High-Precision Determination of the Pion-Nucleon σ Term from Roy-Steiner Equations

Martin Hoferichter,[1,2,3,4] Jacobo Ruiz de Elvira,[5] Bastian Kubis,[5] and Ulf-G. Meißner[5,6]

[1]*Institut für Kernphysik, Technische Universität Darmstadt, D–64289 Darmstadt, Germany*

[2]*ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, D–64291 Darmstadt, Germany*

[3]*Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5, CH–3012 Bern, Switzerland*

[4]*Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1550, USA*

[5]*Helmholtz–Institut für Strahlen- und Kernphysik (Theorie) and Bethe Center for Theoretical Physics, Universität Bonn, D–53115 Bonn, Germany*

[6]*Institut für Kernphysik, Institute for Advanced Simulation, Jülich Center for Hadron Physics, JARA-HPC, and JARA-FAME, Forschungszentrum Jülich, D–52425 Jülich, Germany*

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We present a determination of the pion-nucleon (πN) σ term $\sigma_{\pi N}$ based on the Cheng-Dashen low-energy theorem (LET), taking advantage of the recent high-precision data from pionic atoms to pin down the πN scattering lengths as well as of constraints from analyticity, unitarity, and crossing symmetry in the form of Roy-Steiner equations to perform the extrapolation to the Cheng-Dashen point in a reliable manner. With isospin-violating corrections included both in the scattering lengths and the LET, we obtain $\sigma_{\pi N} = (59.1 \pm 1.9 \pm 3.0)$ MeV $= (59.1 \pm 3.5)$ MeV, where the first error refers to uncertainties in the πN amplitude and the second to the LET. Consequences for the scalar nucleon couplings relevant for the direct detection of dark matter are discussed.

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Introduction.—The πN σ term measures the amount of the nucleon mass that is generated by the two lightest quarks. Since the dominant contribution originates from the energy content of the gluon field, due to the trace anomaly of the QCD energy-momentum tensor, the nucleon mass would only change moderately if the quark masses were turned off. Thus, $\sigma_{\pi N}$ encodes information on the explicit breaking of chiral symmetry and constitutes one of the fundamental low-energy parameters of QCD. In recent years, a precise determination of the σ term has become increasingly urgent, given its relation to the scalar couplings of the nucleon that are prerequisite for a consistent interpretation of direct-detection dark matter searches [1–3].

Traditionally, information on $\sigma_{\pi N}$ has been inferred from πN scattering by means of the Cheng-Dashen low-energy theorem (LET) [4,5] that relates the Born-term-subtracted isoscalar amplitude $\bar{D}^+$ at the Cheng-Dashen point $s = u = m_N^2$, $t = 2M_\pi^2$, to the scalar form factor of the nucleon $\sigma(t)$ evaluated at $t = 2M_\pi^2$ (precise definitions below). The application of the LET thus requires two main ingredients: the analytic continuation of the isoscalar πN amplitude into the unphysical region, and the correction due to the finite momentum transfer in $\sigma(2M_\pi^2)$. The first task has been addressed by extrapolating partial-wave analyses (PWAs) from the physical region to the Cheng-Dashen point by means of dispersion relations [6–8], in particular, in Refs. [9,10] a formalism was developed to express the result of the extrapolation in terms of threshold parameters for πN scattering. Similarly, the scalar form factor requires a dispersive reconstruction to account for the strong $\pi\pi$ rescattering in the isospin-0 S wave [11]. Based on the PWA from Refs. [6,8], a value $\sigma_{\pi N} \sim 45$ MeV was inferred in Ref. [10]. This result was later challenged by a new PWA [12], leading to a much larger value of $\sigma_{\pi N} = (64 \pm 8)$ MeV, although based on the same formalism. In fact, the discrepancy could be traced back, to about equal parts, to different input for the isoscalar πN scattering length, the πN coupling constant, and πN partial waves for the evaluation of the dispersive integrals.

A second strategy that has been pursued relies on chiral perturbation theory (ChPT) to perform the extrapolation to the Cheng-Dashen point. However, to determine low-energy constants still input for the πN phase shifts is required, so that the outcome of the ChPT analyses tends to support the value of $\sigma_{\pi N}$ corresponding to the PWA used as input [13,14]. Moreover, it has been questioned whether the chiral representation is at all accurate enough to permit a reliable extrapolation to the Cheng-Dashen point [15]. For a detailed comparison to results obtained in lattice QCD, we refer to Ref. [16].

In this Letter, we combine two new sources of information on πN scattering that have become available over the last years. First, the measurement of level shifts and decay widths in pionic atoms [17–19] has led to a precision determination of the πN scattering lengths [20,21]. Second, a system of Roy-Steiner (RS) equations has been developed [22] that combines general constraints on the πN scattering amplitude imposed by analyticity, unitarity, and crossing symmetry. The construction proceeds similarly to Roy equations for $\pi\pi$ scattering [23], where the solution for the low-energy phase shifts can be parameterized in terms of the S-wave scattering lengths [24]. In the case of πN scattering, the construction and solution is complicated by the presence of the crossed channel $\pi\pi \to N\bar{N}$, cf. Refs. [25–27], as well as the increased number of relevant partial waves. While partial results have already been presented in Refs. [22,28–30], here we use the complete solution of the RS system to obtain, in combination with the scattering-length constraints from pionic atoms, a precision determination of the πN σ term. In particular, at this level of accuracy the impact of isospin-violating (IV) corrections cannot be ignored, as demonstrated by the isoscalar πN scattering length [20,21,31,32], so that revisiting the Cheng-Dashen LET becomes mandatory.

Cheng-Dashen low-energy theorem.—We start by stating the precise formulation of the Cheng-Dashen LET [4,5]. In the isospin limit, the scattering amplitude for the process

$$\pi^a(q) + N(p) \to \pi^b(q') + N(p'), \tag{1}$$

with pion isospin labels a, b, and Mandelstam variables

$$s = (p+q)^2, \qquad t = (p'-p)^2, \qquad u = (p-q')^2, \tag{2}$$

fulfilling $s+t+u = 2m_N^2 + 2M_\pi^2$, can be expressed as

$$T^{ba}(\nu,t) = \delta^{ba}T^+(\nu,t) + \frac{1}{2}[\tau^b,\tau^a]T^-(\nu,t),$$
$$T^I(\nu,t) = \bar{u}(p')\left\{D^I(\nu,t) - \frac{[\not{q}',\not{q}]}{4m_N}B^I(\nu,t)\right\}u(p), \tag{3}$$

where $\nu = (s-u)/(4m_N)$, $I = \pm$ refers to isoscalar or isovector amplitudes, m_N and M_π to the nucleon and pion mass, τ^a denotes isospin Pauli matrices, and we normalize spinors as $\bar{u}u = 1$. Amplitudes $\mathcal{A}^{I_s}$ with definite s-channel isospin I_s are

$$\begin{pmatrix} \mathcal{A}^{1/2} \\ \mathcal{A}^{3/2} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \mathcal{A}^+ \\ \mathcal{A}^- \end{pmatrix}, \qquad \mathcal{A} \in \{D, B\}. \tag{4}$$

The LET involves the Born-term-subtracted amplitude

$$\bar{D}^+(\nu,t) = D^+(\nu,t) - \frac{g^2}{m_N} - \nu g^2\left(\frac{1}{m_N^2 - s} - \frac{1}{m_N^2 - u}\right), \tag{5}$$

where g is the πN coupling constant. The scalar form factor of the nucleon is defined as the matrix element

$$\sigma(t) = \langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle, \qquad \hat{m} = \frac{m_u + m_d}{2}, \tag{6}$$

with up- and down-quark masses m_u and m_d, momentum transfer $t = (p'-p)^2$, and $\sigma(0) = \sigma_{\pi N}$. The LET then states that

$$\bar{D}^+(0, 2M_\pi^2) = \sigma(2M_\pi^2) + \Delta_R, \tag{7}$$

where Δ_R subsumes higher-order corrections in the chiral expansion. Corrections to the LET have been investigated systematically in $SU(2)$ ChPT, with the result that Δ_R is very small: nonanalytic terms are absent at full one-loop order [15,33], so that the dominant corrections are expected to scale as $M_\pi^2/m_N^2\sigma_{\pi N} \sim 1$ MeV. Indeed, estimating the low-energy constants based on resonance exchange, one obtains [33]

$$|\Delta_R| \lesssim 2\text{ MeV}, \tag{8}$$

an estimate which we will adopt in the following.

In practice, Eq. (7) is usually rewritten as

$$\sigma_{\pi N} = \Sigma_d + \Delta_D - \Delta_\sigma - \Delta_R, \tag{9}$$

where

$$\Delta_\sigma = \sigma(2M_\pi^2) - \sigma_{\pi N}, \qquad \Delta_D = \bar{D}^+(0, 2M_\pi^2) - \Sigma_d,$$
$$\Sigma_d = F_\pi^2(d_{00}^+ + 2M_\pi^2 d_{01}^+). \tag{10}$$

Here, $F_\pi = 92.2$ MeV [34] denotes the pion decay constant and the subthreshold coefficients are defined via the expansion

$$\bar{D}^+(\nu,t) = \sum_{n,m=0}^{\infty} d_{mn}^+ \nu^{2m} t^n. \tag{11}$$

Although individually sizable due to strong $\pi\pi$ rescattering, the difference $\Delta_D - \Delta_\sigma$ was shown to be small in Ref. [11]. Here, we use the updated value [35,36]

$$\Delta_D - \Delta_\sigma = (-1.8 \pm 0.2)\text{ MeV}, \tag{12}$$

which incorporates modern input for $\pi\pi$ phase shifts, effects from $K\bar{K}$ intermediate states, and the uncertainties due to πN parameters.

As alluded to above, the isoscalar channel is known to be sensitive to IV corrections. For this reason, we now derive a version of the LET that takes the dominant IV effects into account. First, we define the σ term as the average value of proton and neutron scalar-current matrix elements ($N \in \{p, n\}$)

$$\sigma_{\pi N} = \frac{\sigma_p + \sigma_n}{2}, \qquad \sigma_N = \langle N|\hat{m}(\bar{u}u + \bar{d}d)|N\rangle, \tag{13}$$

where, up to third order in the chiral expansion, one finds $\sigma_p = \sigma_n$ [37]. Next, we identify the isoscalar amplitudes everywhere with the average of the $\pi^\pm p \to \pi^\pm p$ charge channels

$$X^+ \to X^p = \frac{1}{2}(X_{\pi^+ p\to\pi^+ p} + X_{\pi^- p\to\pi^- p}), \qquad (14)$$

for $X \in \{D, d_{00}, d_{01}, \ldots\}$. The motivation for doing so is twofold: first, the $\pi^\pm p$ charge channels dominate the πN data base, so that this scenario is closest to the one considered in PWAs. Second, the uncertainties in the πN scattering lengths are smallest if one works in the physical, not the isospin basis [20,21]. As a consequence, we identify the nucleon and pion mass with the masses of the proton and the charged pion, respectively. We also assume that the radiative corrections applied in the PWAs remove the dominant effects, and therefore we consider all quantities to be virtual-photon subtracted. In this scenario, the leading IV corrections are generated by the mass difference between charged and neutral pion $\Delta_\pi = M_\pi^2 - M_{\pi^0}^2$. For the scalar form factor one finds [37]

$$\begin{aligned}\Delta_\sigma^p &= \sigma_p(2M_\pi^2) - \sigma_p \\ &= \frac{3g_A^2 M_\pi^3}{64\pi F_\pi^2} + \frac{g_A^2 M_\pi \Delta_\pi}{128\pi F_\pi^2}(-7 + 2\sqrt{2}\log(1+\sqrt{2})),\end{aligned} \qquad (15)$$

where g_A denotes the axial charge of the nucleon. Similarly, the IV corrections to Δ_D can be extracted from [38]

$$\begin{aligned}\Delta_D^p &= F_\pi^2\{D_p(0, 2M_\pi^2) - d_{00}^p - 2M_\pi^2 d_{01}^p\} \\ &= \frac{23 g_A^2 M_\pi^3}{384\pi F_\pi^2} + \frac{g_A^2 M_\pi \Delta_\pi}{256\pi F_\pi^2}(3 + 4\sqrt{2}\log(1+\sqrt{2})).\end{aligned} \qquad (16)$$

Taking everything together, we obtain

$$\begin{aligned}\sigma_{\pi N} &= F_\pi^2(d_{00}^p + 2M_\pi^2 d_{01}^p) + \Delta_D - \Delta_\sigma - \Delta_R \\ &\quad + \frac{81 g_A^2 M_\pi \Delta_\pi}{256\pi F_\pi^2} + \frac{e^2}{2}F_\pi^2(4f_1 + f_2) \\ &= F_\pi^2(d_{00}^p + 2M_\pi^2 d_{01}^p) + (1.2 \pm 3.0)\text{ MeV}.\end{aligned} \qquad (17)$$

In Eq. (17) we also included the leading corrections due to virtual photons, encoded in the low-energy constants f_1 and f_2. The latter can be determined from the proton-neutron mass difference [39], $f_2 = (-0.97 \pm 0.38)\text{ GeV}^{-1}$, for the former we use the estimate $|f_1| \le 1.4\text{ GeV}^{-1}$ [31,40]. The single largest correction is generated by Δ_π, an upward shift of 3.4 MeV. Such large IV corrections have already been observed in the case of the πN scattering lengths [31,32].

Pionic atoms.—Pionic hydrogen (πH) and deuterium (πD), a π^- and a proton/deuteron bound by electromagnetism, provide access to πN physics due to the imprint of strong interactions in the energy spectrum. The shift of the ground-state energy level in πH and πD, as well as the width of the πH ground state, probe three different combinations of πN scattering lengths. The input quantities relevant for the RS equations are the s-channel-isospin scattering lengths $a_{0+}^{I_s}$, defined in terms of the $\pi^\pm p$ charge channels. Updating the analysis of Refs. [20,21] to account for the new value of the πH level shift [19] and subtracting virtual-photon effects as detailed in Ref. [41], we obtain

$$\begin{aligned}a_{0+}^{1/2} &= (169.8 \pm 2.0) \times 10^{-3} M_\pi^{-1}, \\ a_{0+}^{3/2} &= (-86.3 \pm 1.8) \times 10^{-3} M_\pi^{-1}.\end{aligned} \qquad (18)$$

Since the errors are dominated by different sources—IV corrections in the case of $a_{0+}^{1/2}$ and uncertainty in the extraction of the isoscalar combination for $a_{0+}^{3/2}$—the errors can be considered uncorrelated to a very good approximation. Apart from their role in the solution of the RS equations, the scattering lengths are also a crucial ingredient in the determination of the πN coupling constant via the Goldberger-Miyazawa-Oehme sum rule [42]. Indeed, if the scattering lengths from Refs. [6,8] are used, one recovers the value $g^2/(4\pi) = 14.3$, whereas Eq. (18) leads to $g^2/(4\pi) = 13.7 \pm 0.2$ [20,21]. This result, to be adopted in the following, stands in good agreement with more recent determinations from NN [43] and πN [44] scattering.

Roy-Steiner equations.—Roy equations [23] for $\pi\pi$ scattering, or RS equations [22,25–27] for nontotally-crossing-symmetric processes, incorporate the constraints from analyticity, unitarity, and crossing symmetry in the form of dispersion relations for the partial waves. They can be shown to be rigorously valid in a certain kinematic region, in the case of πN scattering the upper limit is $s_\mathrm{m} = (1.38\text{ GeV})^2$ [22]. The integral contributions above s_m as well as partial waves with $l > l_\mathrm{m}$, with l_m the maximal angular momentum explicitly included in the calculation, are collected in the so-called driving terms, which need to be estimated from existing PWAs, as do inelastic contributions below s_m. The free parameters of the approach are subtraction constants, which, in the case of $\pi\pi$ scattering, can be directly identified with the scattering lengths [24], while for the solution of the πN system it is more convenient to relate them to subthreshold parameters instead. The resulting system of coupled integral equations corresponds to a self-consistency condition for the low-energy phase shifts, whose mathematical properties were investigated in detail in Ref. [45]. Following Ref. [24], we pursue the following solution strategy: the phase shifts are parameterized in a convenient way with a few parameters each, which are matched to input partial waves above s_m in a smooth way. To measure the degree to which the RS are fulfilled, a χ^2-like function is defined according to

$$\chi^2 = \sum_{l,I_s,\pm}\sum_{j=1}^{N}\left(\frac{\mathrm{Re} f_{l\pm}^{I_s}(W_j) - F[f_{l\pm}^{I_s}](W_j)}{\mathrm{Re} f_{l\pm}^{I_s}(W_j)}\right)^2, \qquad (19)$$

where $\{W_j\}$ denotes a set of points between threshold and $\sqrt{s_\mathrm{m}}$, $f_{l\pm}^{I_s}$ are the s-channel partial waves with isospin I_s, orbital angular momentum l, and total angular momentum $j = l \pm 1/2 \equiv l\pm$, and $F[f_{l\pm}^{I_s}]$ the right-hand side of the RS equations. We take $l_{\mathrm{m}}=1$, $N=25$ (distributed equidistantly), and choose the number of subtraction constants in such a way as to match the number of degrees of freedom predicted by the mathematical properties of the Roy equations [45]. It should be stressed that the form of the RS equations only reduces to that of Roy equations once the t-channel is solved, see Ref. [22]. In the solution of the RS equations we minimize Eq. (19) with respect to the subtraction constants (identified with subthreshold parameters) and the parameters describing the low-energy phase shifts, while imposing Eqs. (18) as additional constraints.

We performed a number of checks as regards the sensitivity of our solution to the input quantities: the number of grid points N as well as the number of parameters used in the description of the partial waves were varied, the s- and t-channel partial waves in the driving terms truncated at different $l_{\max}=4,5$ and $J_{\max}=2,3$, the matching conditions at s_{m} as well as the s-channel partial waves evaluated from different PWAs, and the sensitivity to the precise definition of the χ^2 function was investigated. In addition, to stabilize the fit we imposed sum rules for the higher subthreshold parameters. The solution for the s-channel partial waves, expressed in terms of the phase shifts and including uncertainty estimates from these systematic studies as well as the uncertainties in the scattering lengths and the coupling constant, is shown in Fig. 1. A more detailed account of our RS solution will be given in Ref. [16].

Apart from low-energy phase shifts, the RS solution provides a consistent set of subthreshold parameters. In particular, this allows us to pin down Σ_d in accord with both the RS and the scattering-length constraints. Linearizing around the central values [Eq. (18)], we find

$$\Sigma_d=(57.9\pm0.9)\ \mathrm{MeV}+\sum_{I_s}c_{I_s}\Delta a_{0+}^{I_s},$$
$$c_{1/2}=0.24\ \mathrm{MeV},\qquad c_{3/2}=0.89\ \mathrm{MeV},\qquad (20)$$

where $\Delta a_{0+}^{I_s}$ measures the deviation from Eq. (18) in units of $10^{-3}M_\pi^{-1}$. Already in this linearized form, one recovers Σ_d from Eq. (10) if the scattering lengths from Refs. [6,8] are inserted, while the modern input produces $\Sigma_d=(57.9\pm1.9)$ MeV (this also indicates that the S-wave phase shifts from Refs. [6,8] need to be amended close to threshold). Moreover, the difference to Ref. [12] can be traced back to the P-wave scattering volume a_{1+}^{+}, which needs to be known extremely accurately due to its large weight in the formalism of Refs. [9,10]. Once the RS equations are solved, the threshold parameters can be calculated from sum rules, and indeed we find that the result for a_{1+}^{+} is slightly lower than the value used in Ref. [12], which already suffices to explain the difference [16]. The main impact of the RS equations in the σ-term determination thus amounts to eliminating the need for independent input for a_{1+}^{+}. In total, our result for the σ term becomes

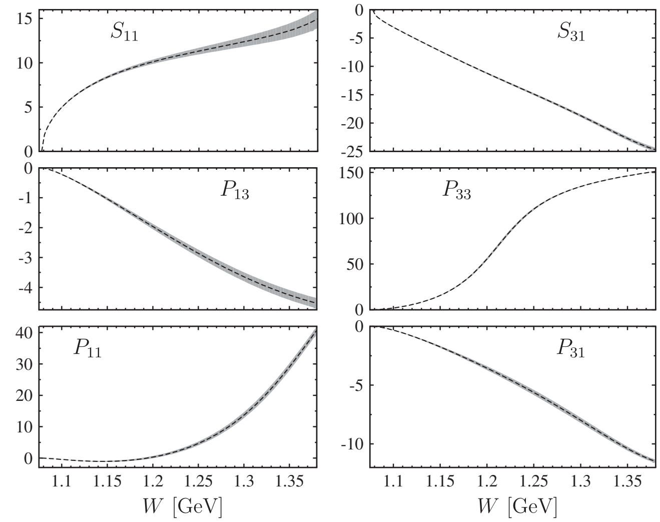

FIG. 1. Phase shifts $\delta_{l\pm}^{I_s}$ of the s-channel partial waves in degrees, obtained from the solution of the RS equations. The dashed line indicates our central solution, the bands the uncertainty estimate. The partial waves are labeled by the spectroscopic notation $L_{2I_s 2J}$.

$$\sigma_{\pi N}=(59.1\pm3.5)\ \mathrm{MeV}.\qquad (21)$$

Although already 4.2 MeV are due to new corrections to the LET (thereof 3.0 MeV from isospin breaking), we do observe a significant increase compared to Ref. [10]. As illustrated by Eq. (20), this effect can be immediately traced back to our modern knowledge of the πN scattering lengths as extracted from pionic atoms. By combining this information with the constraints from RS equations, the σ term can be determined to a remarkable accuracy.

Scalar nucleon couplings.—The existence of a weakly-interacting massive particle (WIMP), one of the most promising dark-matter candidates, could be established in direct-detection experiments, which are sensitive to the recoil of the WIMP scattering off nuclei (see Ref. [46] for a review). The interpretation of these searches relies on the couplings of the WIMP to nucleons, according to its quantum numbers. A precise determination of $\sigma_{\pi N}$ therefore has immediate consequences for the scalar channel, since, as it was shown in Ref. [3], the scalar couplings of the nucleon to $q=u,d$,

$$m_N f_q^N=\langle N|m_q\bar{q}q|N\rangle,\qquad (22)$$

follow once $\sigma_{\pi N}$ is determined, with all further corrections taken into account within $SU(2)$ ChPT. Taking $m_u/m_d=0.46\pm0.03$ from Ref. [47], we obtain

$$f_u^p=(20.8\pm1.5)\times10^{-3},\qquad f_d^p=(41.1\pm2.8)\times10^{-3},$$
$$f_u^n=(18.9\pm1.4)\times10^{-3},\qquad f_d^n=(45.1\pm2.7)\times10^{-3}.\qquad (23)$$

In addition, we quote our result for

$$\sum_{q=u,\ldots,t} f_q^N = \frac{2}{9} + \frac{7}{9}(f_u^N + f_d^N + f_s^N) = 0.305 \pm 0.009, \tag{24}$$

averaged over the proton and neutron, and with f_s^N taken from Ref. [48] (in principle, the strangeness coupling follows from the σ term by means of $SU(3)$ considerations, but the uncertainties are too large to compete with recent lattice determinations). This particular combination of scalar coefficients becomes relevant in the context of Higgs-mediated interactions, not only in direct detection, but also in Higgs-induced lepton flavor violation [49]. In particular in Eq. (23) the uncertainties have been appreciably reduced, thanks to the precise knowledge of $\sigma_{\pi N}$ inferred from our RS equation analysis of πN scattering.

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